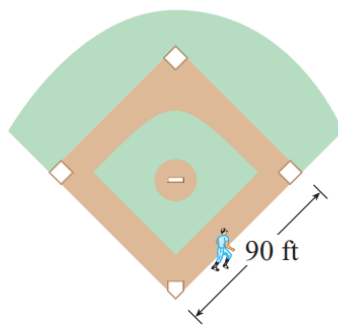
Exercise 20

A baseball diamond is a square with side 90 ft. A batter hits the ball and runs toward first base with a speed of 24 ft/s.

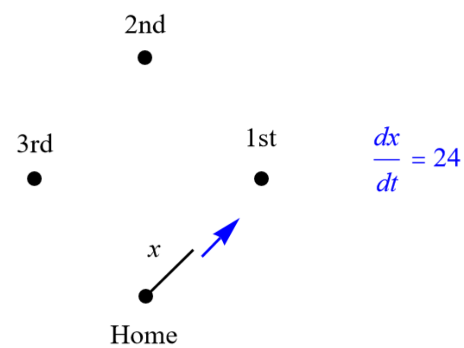
(a) At what rate is his distance from second base decreasing when he is halfway to first base?

(b) At what rate is his distance from third base increasing at the same moment?



Solution

Let x be the batter's distance from home base. Then dx/dt is the speed that he's running.

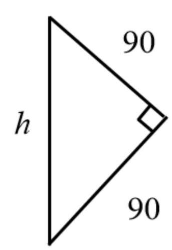


Part (a)

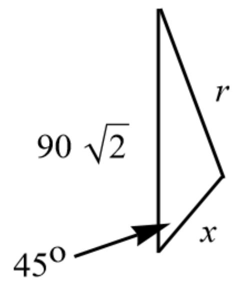
Start by finding the distance from home base to second base using the Pythagorean theorem.

$$h^2 = 90^2 + 90^2$$

$$h = 90\sqrt{2}$$



At any time, the distance from the batter to second base r is related to x by the law of cosines.



$$r^2 = (90\sqrt{2})^2 + x^2 - 2(90\sqrt{2})(x)\cos 45^\circ$$

$$= 16\,200 + x^2 - 2(90\sqrt{2})(x)\left(\frac{1}{\sqrt{2}}\right)$$

$$= 16\,200 + x^2 - 180x$$

Take the derivative of both sides with respect to t and use the chain rule.

$$\frac{d}{dt}(r^2) = \frac{d}{dt}(16\,200 + x^2 - 180x)$$

$$2r \cdot \frac{dr}{dr} = 2x \cdot \frac{dx}{dt} - 180 \cdot \frac{dx}{dt}$$

$$r\frac{dr}{dr} = x\frac{dx}{dt} - 90\frac{dx}{dt}$$

$$= (x - 90)\frac{dx}{dt}$$

$$= (x - 90)24$$

Solve for dr/dt.

$$\frac{dr}{dt} = \frac{24(x-90)}{r}$$

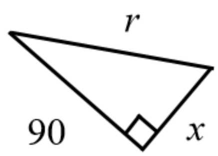
$$= \frac{24(x-90)}{\sqrt{16\,200 + x^2 - 180x}}$$

When the batter is halfway to first base, $x = 45$. Therefore, the desired rate is

$$\left.\frac{dr}{dt}\right|_{x=45} = \frac{24(45-90)}{\sqrt{16\,200 + (45)^2 - 180(45)}} = -\frac{24}{\sqrt{5}} \approx -10.7331\ \frac{\text{ft}}{\text{s}}.$$

Part (b)

At any time, the distance from the batter to third base r is related to x by the Pythagorean theorem.



$$r^2 = 90^2 + x^2$$

Take the derivative of both sides with respect to time using the chain rule.

$$\frac{d}{dt}(r^2) = \frac{d}{dt}(90^2 + x^2)$$

$$2r \cdot \frac{dr}{dt} = 2x \cdot \frac{dx}{dt}$$

$$r\frac{dr}{dt} = x\frac{dx}{dt}$$

$$= x(24)$$

Solve for dr/dt.

$$\frac{dr}{dt} = \frac{24x}{r}$$

$$= \frac{24x}{\sqrt{90^2 + x^2}}$$

When the batter is halfway to first base, $x = 45$. Therefore, the desired rate is

$$\left.\frac{dr}{dt}\right|_{x=45} = \frac{24(45)}{\sqrt{90^2 + (45)^2}} = \frac{24}{\sqrt{5}} \approx 10.7331 \ \frac{\text{ft}}{\text{s}}.$$